

## ECE311 Quiz 2

**Note: Use of a calculator is not permitted for this quiz.**

**Problem:** Determine the number of 1) LHP (left half plane), 2) RHP (right half plane), and 3) IA (imaginary axis) roots for each of the following polynomials using the Routh-Hurwitz criteria.

1)  $s^3 + s^2 + 2s + 8$

2)  $s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$

3)  $s^4 + s^3 + 3s^2 + 2s + 2$

# SOLUTIONS

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3)  $s^4 + s^3 + 3s^2 + 2s + 2$

a)  $s^3 + s^2 + 2s + 8$

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 1 & 8 \\ s^1 & -6 & 0 \\ s^0 & 8 & \end{array}$$

2 SIGN CHANGES  $\Rightarrow$

# RHP = 2

$\Rightarrow$  # LHP =  $3 - 2 = 1$

$\Rightarrow$  # IA = 0

NOTE: USING MATLAB

ROOTS ARE

-2,  $\leftarrow$  1xLHP

$0.5 \pm 1.94j$

$\leftarrow$  2xRHP

b)  $s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$

$$\begin{array}{c|ccc} s^5 & 1 & 2 & 11 \\ s^4 & 2 & 4 & 10 \\ s^3 & 0 & 6 & \leftarrow \\ s^2 & & & \\ s^1 & & & \\ s^0 & & & \end{array}$$

REPLACE THIS ROW WITH

$$\begin{array}{cc} 0 & 6 \\ -6 & 0 \\ \hline -6 & 6 \end{array}$$

$$\Rightarrow \begin{array}{c|ccc} s^5 & 1 & 2 & 11 \\ s^4 & 2 & 4 & 10 \\ s^3 & -6 & 6 & 0 \\ s^2 & 6 & 10 & \\ s^1 & 16 & 0 & \\ s^0 & 10 & & \end{array}$$

# SIGN CHANGES

$\Rightarrow$  # RHP = 2

$\Rightarrow$  # LHP =  $5 - 2 = 3$

# IA = 0

c)  $s^4 + s^3 + 3s^2 + 2s + 2$

$$\begin{array}{c|ccc} s^4 & 1 & 3 & 2 \\ s^3 & 1 & 2 & \\ s^2 & 1 & 2 & \\ s^1 & 0 & & \\ s^0 & & & \end{array}$$

2x IA ROOTS

$\leftarrow$  Row of ZEROS.  
REPLACE WITH DERIVATIVE  
OF DIVISOR POLYNOMIAL

$P(s) = s^2 + 2$

$\frac{dP(s)}{ds} = 2s$

$\Rightarrow$

$$\begin{array}{c|ccc} s^4 & 1 & 3 & 2 \\ s^3 & 1 & 2 & \\ s^2 & 1 & 2 & \\ s^1 & 2 & 0 & \\ s^0 & 2 & & \end{array}$$

$\Rightarrow$  NO SIGN CHANGES

$\Rightarrow$  # RHP = 0

# IA = 2

$\Rightarrow$  # LHP =  $4 - 2 = 2$

USING MATLAB ROOTS ARE:

$\pm 1.4142j$   $\leftarrow$  2x IA

$-0.5 \pm 0.966j$   $\leftarrow$  2xLHP

PROBLEM  
b. USING MATLAB

ROOTS ARE:

$0.895 \pm 1.46j$   $\leftarrow$  2xRHP

$-1.24 \pm 1.04j$   $\leftarrow$  3xLHP

$-1.31$

$$\begin{vmatrix} 2 & 4 \\ -6 & 6 \end{vmatrix} = \frac{12 + 24}{6} = \frac{36}{6} = 6$$

$$\begin{vmatrix} 2 & 10 \\ -6 & 0 \end{vmatrix} = 10$$

$$\begin{vmatrix} -6 & 6 \\ 6 & 10 \end{vmatrix} = \frac{-60 - 36}{-6} = \frac{96}{6} = 16$$